**Assignment 3\_systembiology**

**Case (i): All Variables are Mutually Independent**

1. The joint probability is simply the product of the individual probabilities: P(A,B,C,D,E)=P(A)×P(B)×P(C)×P(D)×P(E). To fully specify this distribution, we need to know the individual probabilities of A, B, C, D, and E. That's a total of 5 parameters, one for each variable.

Case ii

* **E** depends on **A**, **C**, and **D**.
* **C** depends on **A**.
* **D** depends on **A** and **B**.

So the joint probability distribution P(A,B,C,D,E)is: P(A,B,C,D,E)=P(A)×P(B)×P(C∣A)×P(D∣A,B)×P(E∣A,C,D)

And here's the correct parameter count based on the dependencies:

* P(A) and P(B) are independent, hence we need 1 parameter for each, assuming binary variables.
* P(C∣A) requires parameters for each state of A. With binary A, we need 2 parameter (since the sum of probabilities for all states of C given A must be 2).
* P(D∣A,B)P(D∣A,B) requires parameters for each combination of states of A and B. For binary A and B, this would be 2 \* 2 = 4 combinations.
* P(E∣A,C,D) requires parameters for each combination of states of A, C, and D. If all three are binary, this results in 2 \* 2 \* 2 = 8 combinations, and we subtract one parameter because the probabilities
* 1 for P(A)P(A)
* 1 for P(B)P(B)
* 2 for P(C∣A)P(C∣A)
* 4 for P(D∣A,B)P(D∣A,B)
* 8 for P(E∣A,C,D)P(E∣A,C,D)

Adding these up, we get a total of 16 parameters for this Bayesian network.

**For (ii) P(A=ON,B=ON,C=ON,D=ON,E=ON):**

Based on the network structure and the probabilities provided:

1. P(A=ON)=0.6
2. P(B=ON∣A=ON)=0.95
3. P(C=ON∣A=ON)=0.5
4. P(D=ON∣A=ON,B=ON)=0.95
5. P(E=ON∣D=ON)=0.1

The joint probability is the product of these probabilities, since we are considering the case where all variables are "ON":

P(A=ON,B=ON,C=ON,D=ON,E=ON)= P(A=ON)×P(B=ON∣A=ON)×P(C=ON∣A=ON)×P(D=ON∣A=ON,B=ON)×P(E=ON∣D=ON)

Now let's plug in the values:

P(A=ON,B=ON,C=ON,D=ON,E=ON)=0.6×0.95×0.5×0.95×0.1

The probability of P(A=ON,B=ON,C=ON,D=ON,E=ON) given Network 2 and the provided conditional probabilities is approximately = **0.0271**.

Ii P(E = ON | A = ON)

The probability of P(A=ON,B=ON,C=ON,D=ON,E=ON)P(A=ON,B=ON,C=ON,D=ON,E=ON) given Network 2 and the provided conditional probabilities is approximately 0.0271.

**For (ii) P(E=ON∣A=ON)**

To calculate this conditional probability, we need to consider the ancestor subgraph of E when A is known to be ON.

In Network 2, E depends only on D, and D depends on A and B. Since A is given to be ON, we only need the conditional probabilities of D given A and B, and then E given D.

However, to find P(E=ON∣A=ON), we don't need the values of B or C since they are not ancestors of E. Hence, we need to sum over all possible values of D, but since we only have binary states, we can directly

P(E=ON∣A=ON)=∑B,D ​P(E=ON∣D=ON)⋅P(D=ON∣A=ON,B)⋅P(B∣A=ON)

We have two conditions for B: B = ON and B = OFF. We will calculate the probability for each and sum them:

When B=ON: P(E=ON∣D=ON)⋅P(D=ON∣A=ON,B=ON)⋅P(B=ON∣A=ON)

=0.1x 0.95 x0.95= 0.9025

When B=OFF: P(E=ON∣D=ON)⋅P(D=ON∣A=ON,B=OFF)⋅P(B=OFF∣A=ON) =0.1x 0.3x (1−0.95)= 0.0015

Adding these two probabilities gives us P(E=ON∣A=ON)P:

Then we will sum the products of these probabilities with P(E=ON∣D=ON) and P(E=ON∣D=OFF)= 0.1X

0.8 = 0.9

P(E=ON∣A=ON)=0.09025+0.0015+0.9 = 0.99175

Thus, the probability of (E=ON|A=ON) given that A=ON is 0.99175.